WHAT TO DO WHEN THINGS GET VERY SMALL OR THE PORT IS JUST TOO BIG FOR THE BOX

BY STEVE MOWRY

Typically, in a small woofer design the transducer and/or loudspeaker engineer will increase the force factor, **Bl** (Tm) and the moving mass, **M** (kg) to reduce the first natural frequency, $f_o = \frac{1}{2\pi}\sqrt{\frac{K}{M}}$ (Hz) and increase the motor force. The increasing force and mass is an attempt to improve the low-end performance of a woofer but it can be observed that the affects of these parameters in a small box application tend to go in opposite directions. Once the box becomes so small that it dominates the loudspeaker's stiffness, **K** (N/m) and first natural frequency, f_o, it seems counterproductive to increase mass in such an application. However, on the other hand, with prices of Neodymium Iron Boron magnets at an all time low and steel prices up, it makes sense to rethink motor designs and the **Bl** or more important the $\beta = [(Bl)^2]/R_e$ (N^2/W) requirements. R_e is the DC resistance of the voice coil (Ω). These quantities can be increased without a significant cost impact and in some cases, a cost reduction. Using the new grades of Neodymium Iron Boron magnets makes implementing a modern high β woofer in a small sealed box with a reasonably high power amplifier, a relatively simple and cost effective design option.

When a woofer is placed in a small box such that the net volume of air within the enclosure, $V_{box} << V_{as}$ (m^3), then the results are obvious. The first natural frequency of the woofer within the small box will be $f_{box} >> f_o$. There is no way around this and please remember that as f_o is decreased, V_{as} is increased; get the point, ouch? The small volume of trapped air within the box, K_{box} is a stiff spring and dominates the loudspeaker's stiffness, $K = K_{box} + K_{ms}$, where $K_{ms} = K_{spider} + K_{surround}$. The box can be ported and tuned to resonate at a frequency below f_{box}; however, that port will be long and if not properly designed, it will be noisy. Furthermore, such a port will displace significant volume within the enclosure. In such situations, a port is a win-lose at best with some serious inherent design challenges.

The equalized sealed box design approach uses the amplifier to drive the moving assembly harder as frequency is reduced. Everything is moving as a piston at these frequencies. A ported box depends on a port/enclosure resonance to augment low frequency output. Actually these approaches appear similar when illustrated as a system; however, the equalized sealed box is implemented in the electrical domain while the ported box is implemented in the acoustic domain. Both the equalized sealed box and the ported box are 4^{th} order systems, the same number of complex "poles" in the loudspeaker transfer function, four.

Thiele-Small Parameter

Re	3.20	Ohm
Le	3.00	mH
Reb	undef.	Ohm
Leb	undef.	mH
fs	52.6	Hz
Qms	4.00	
Qes	0.40	
Qts	0.36	
Rms	1.01	Ns/m
Mmd	11.82	g
Mms	12.20	g
Cms	7.5e-04	m/N
Cas	4.4e-08	m^5/N
Vas	6.23	l
Sd	77.0	cm^2
Bl	5.70	N/A

FIGURE 1. THE WOOFER MODEL, $\boldsymbol{f_o}$ = 52 Hz.

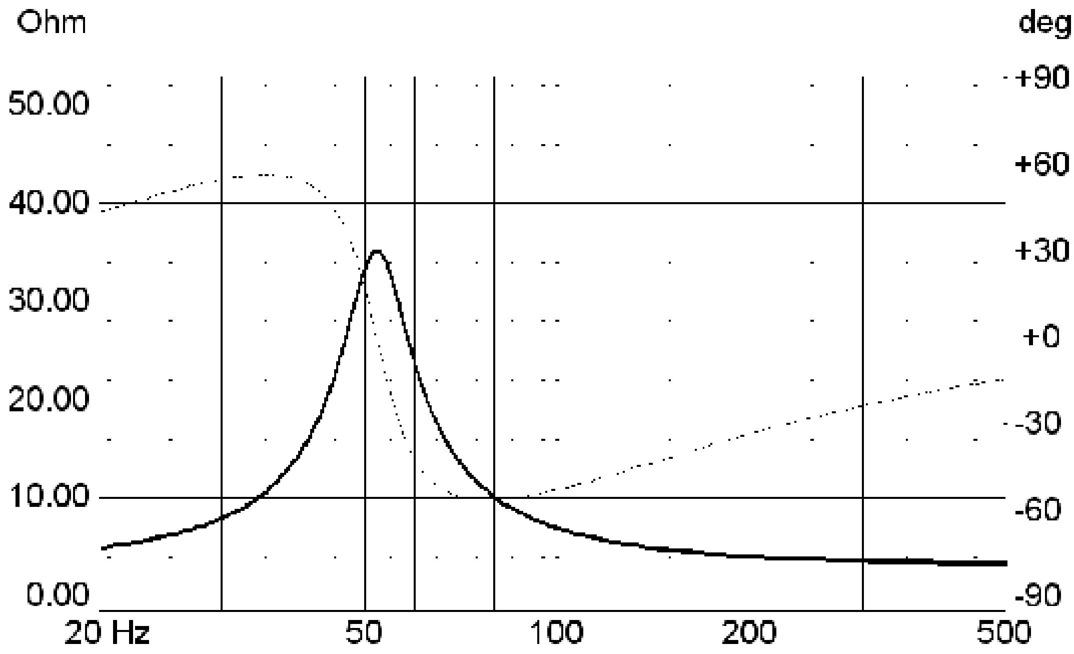

FIGURE 2. PLOT OF IMPEDANCE, $\boldsymbol{Z(f)}$ OF THE 5 INCH WOOFER MOUNTED IN A 10 l BOX, $\boldsymbol{f_{box}} \approx \boldsymbol{f_o}$.

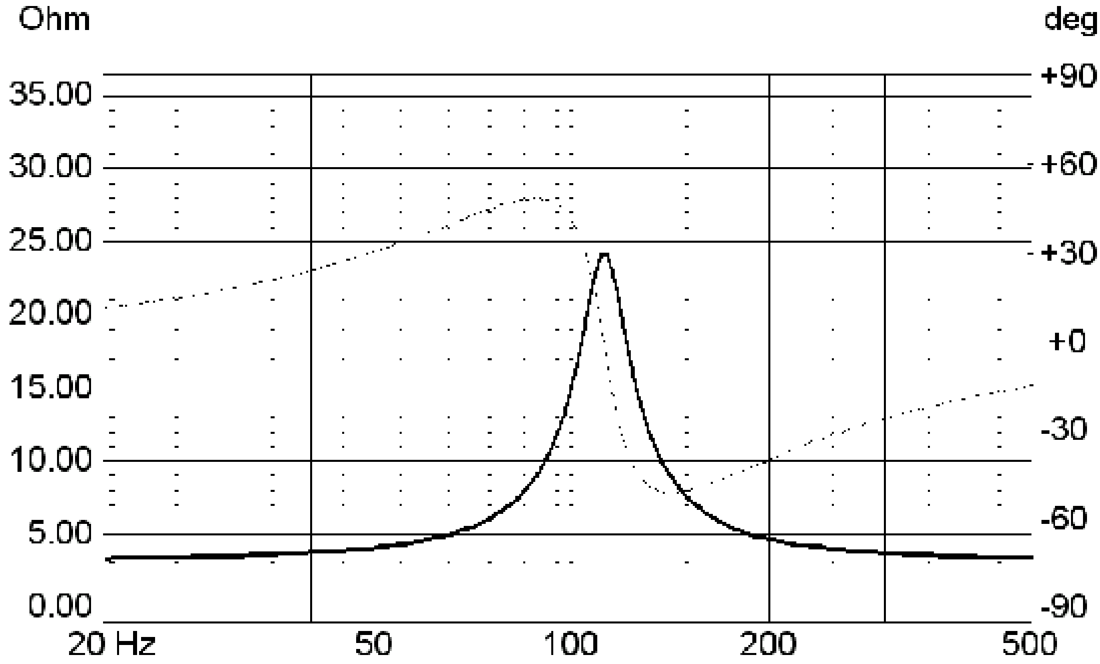

FIGURE 3. PLOT OF IMPEDANCE, **Z(*f*)** OF THE SAME WOOFER MOUNTED IN A 1.8 l SEALED BOX, $\boldsymbol{f_{box} \approx 2f_o}$.

Figures 2 and 3 illustrates that the small box is dominating the stiffness and the natural frequency.

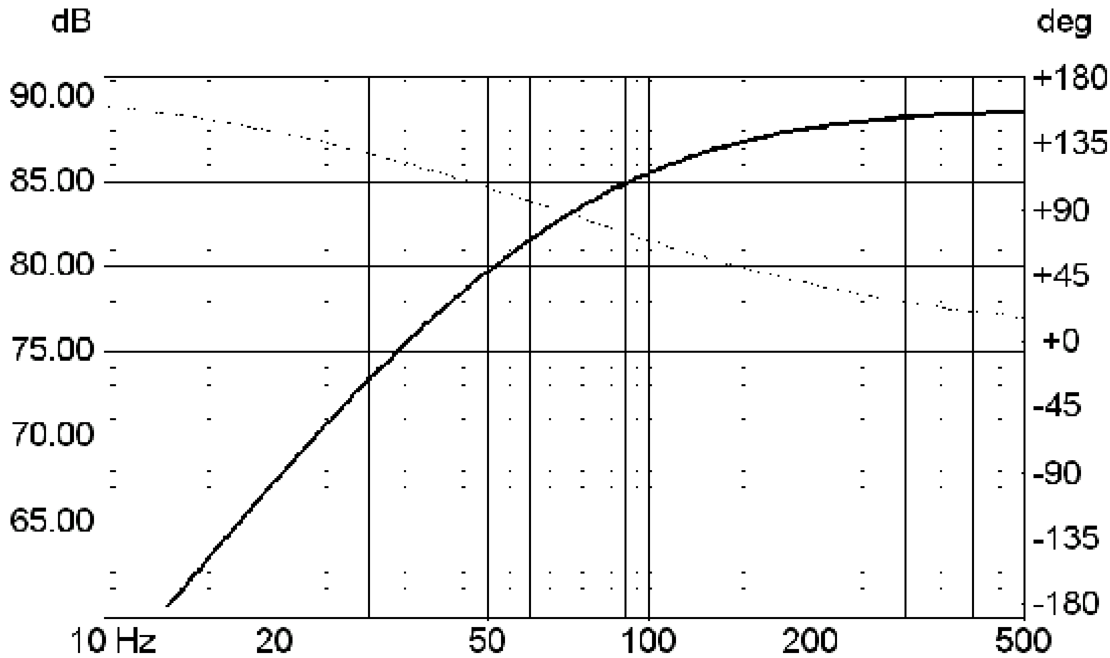

FIGURE 4. SIMULATED FREQUENCY RESPONSE WITH 2.83 V INPUT.

Thiele-Small Parameter

Re	3.20	Ohm
Le	3.00	mH
Reb	undef.	Ohm
Leb	undef.	mH
fs	26.3	Hz
Qms	4.00	
Qes	0.20	
Qts	0.19	
Rms	0.50	Ns/m
Mmd	11.82	g
Mms	12.20	g
Cms	3.0e-03	m/N
Cas	1.8e-07	m^5/N
Vas	24.90	l
Sd	77.0	cm^2
Bl	5.70	N/A

FIGURE 5. LOW STIFFNESS WOOFER, $\boldsymbol{f_o}$ = 26 Hz.

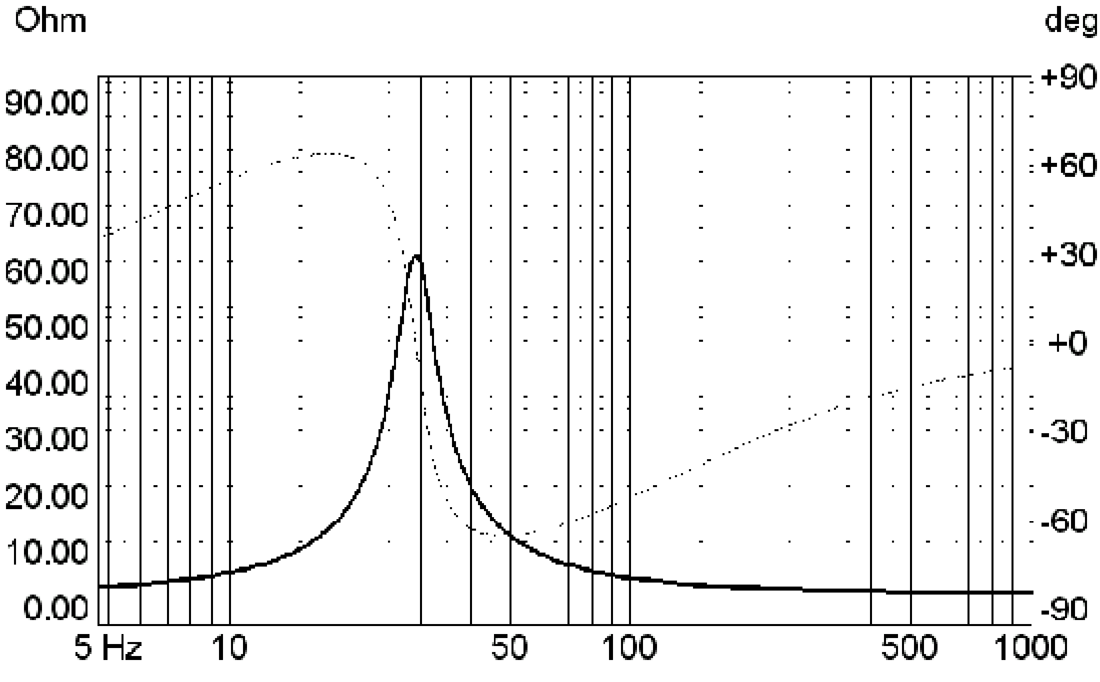

FIGURE 6. IMPEDANCE CURVE OF LOW STIFFNESS WOOFER IN A LARGE BOX, 100 l. $\boldsymbol{f_{box}} > \boldsymbol{f_o}$

Figure 6 illustrates that $\mathbf{V_{as}}$ for the 26 Hz woofer is large. Even with a 100 liter box, f_{box} is greater than $\boldsymbol{f_o}$.

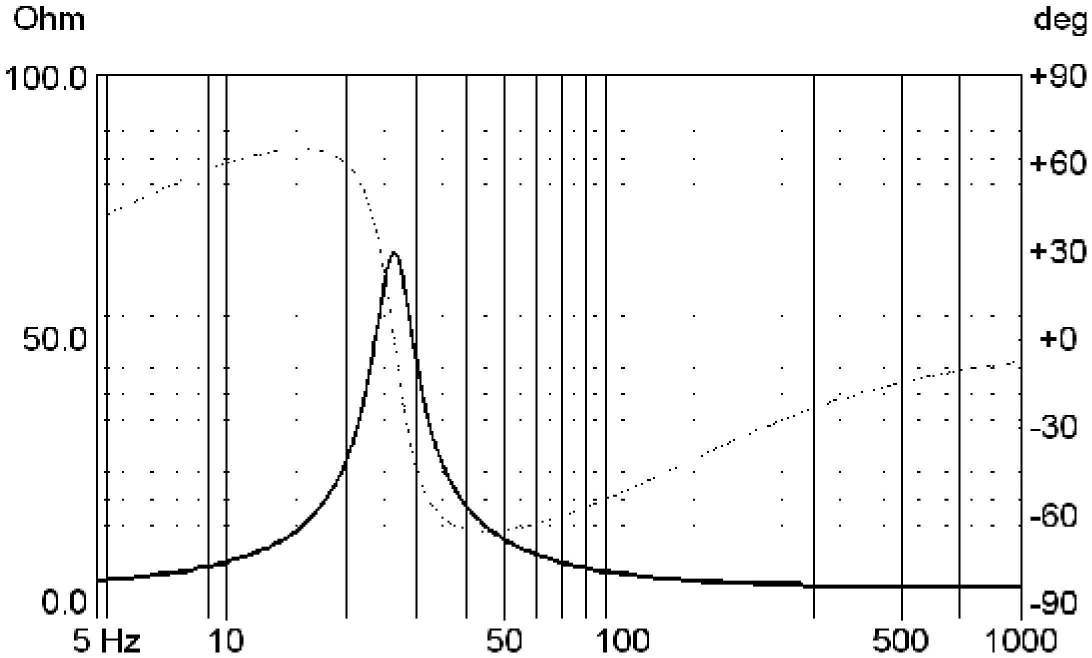

FIGURE 7. IMPEDANCE CURVE OF LOW STIFFNESS WOOFER IN AN INFINITE BOX, 1000 l, $\boldsymbol{f}_{\mathbf{box}} = \boldsymbol{f}_{\mathbf{o}}$

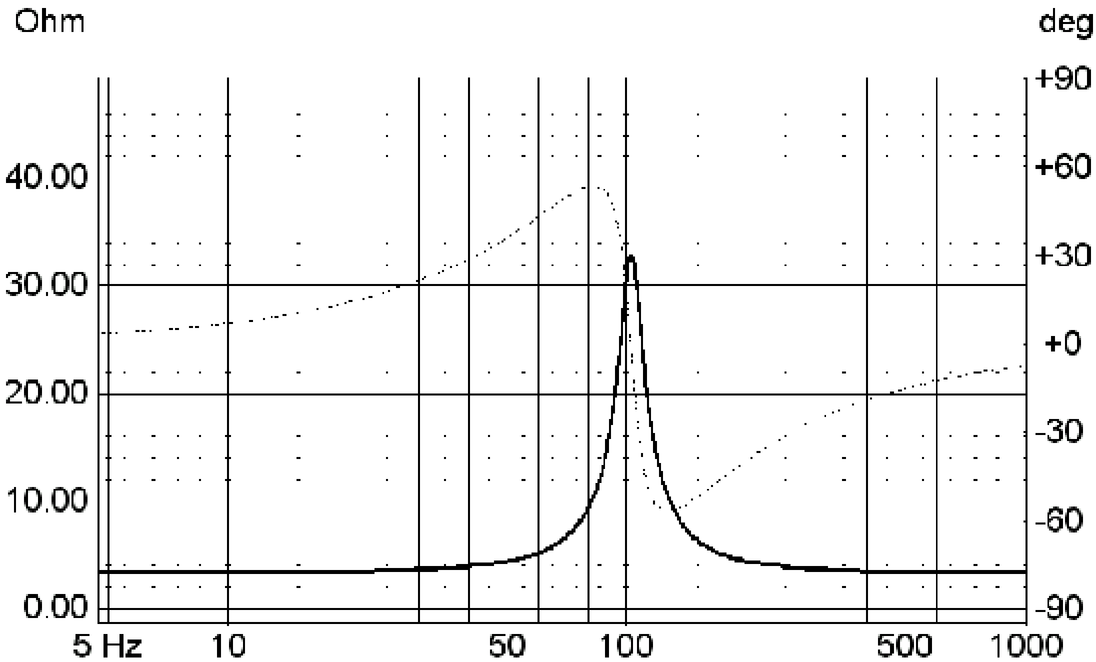

FIGURE 8. IMPEDANCE CURVE FOR THE SAME WOOFER AS IN FIGURE 7 BUT MOUNTED IN A 1.8 l BOX, $\boldsymbol{f}_{\mathbf{box}} \approx \mathbf{4}\boldsymbol{f}_{\mathbf{o}}$.

By cutting $\boldsymbol{f}_{\mathbf{o}}$ for the 5.25-inch woofer in half, from 52.6 Hz to 26.3 Hz; that's a 4 fold reduction in the woofer's stiffness, the net reduction in $\boldsymbol{f}_{\mathbf{box}}$ is only reduced by approximately 10 Hz! Please compare figures 3 and 8. This is not mentioning that a robust 26 Hz resonance 5.25 inch will be very difficult to realize anyway. The point here is that the box stiffness will dominate and adding mass will only reduce pass band sensitivity and increase mechanical issues including high-pressure gradient on the surround at the tuning frequency of a ported box design. Then a low mass moving assembly with an $\boldsymbol{f_o}$ = 50 Hz is a reasonable target for a 5.25 inch woofer in a 1.75 L box.

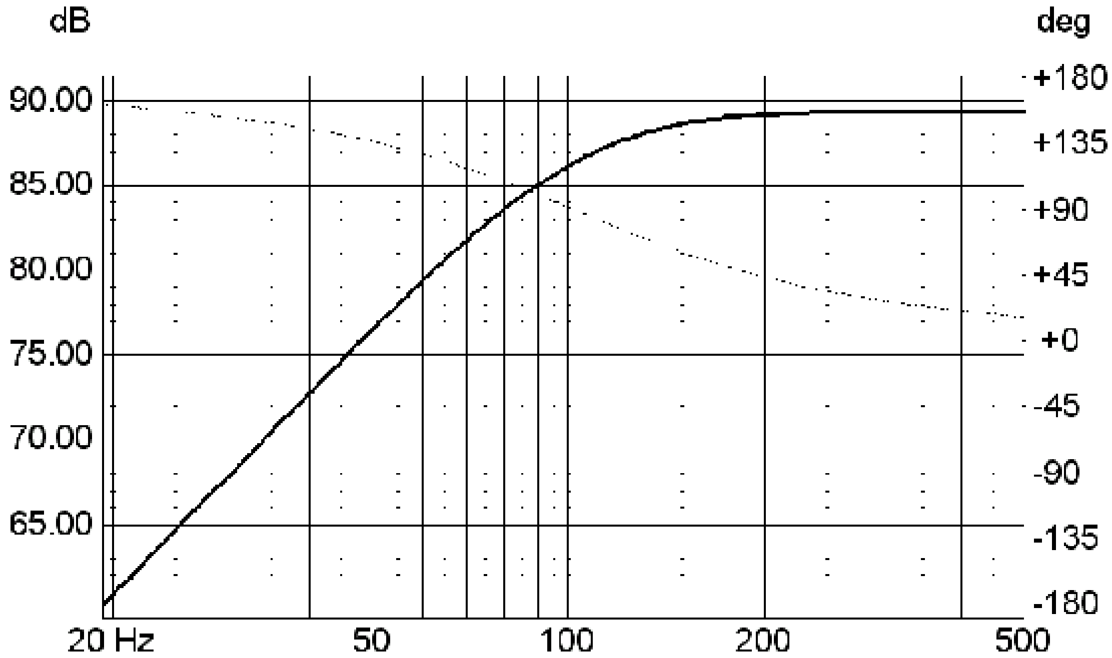

FIGURE 9. SIMULATED FREQUENCY RESPONSE WITH 2.83 V INPUT.

Looking at the small signal simulated frequency response plots illustrated in figures 4 and 9, it is obvious that there is a lack of low frequency amplitude with the systems using both the 52 Hz woofer and the 26 Hz woofer in a 1.75 liter sealed box. There is very little difference between these two loudspeakers with respect to frequency response, ~ 0.5 dB at 100 Hz. This is not even audible and is a much less than production tolerance.

Please don't despair. There is an elegantly simple alternative but with some sacrifice in pass band output verses low-end cut-off is the equalized sealed box. This system uses just one 2nd order under-damped high-pass filter just above the low-end cut-off of the loudspeaker. Increasing the Q, quality factor and moving cut-off frequency of the filter about some target can then adjust the system response.

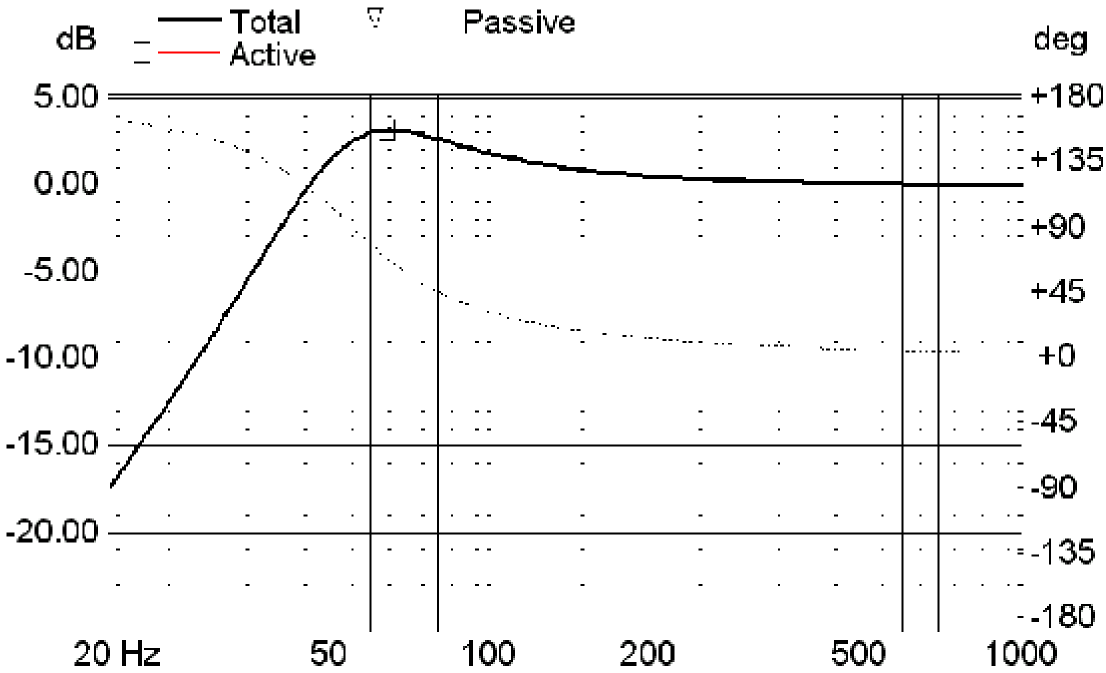

FIGURE 10. THE HP FILTER'S RESPONSE.

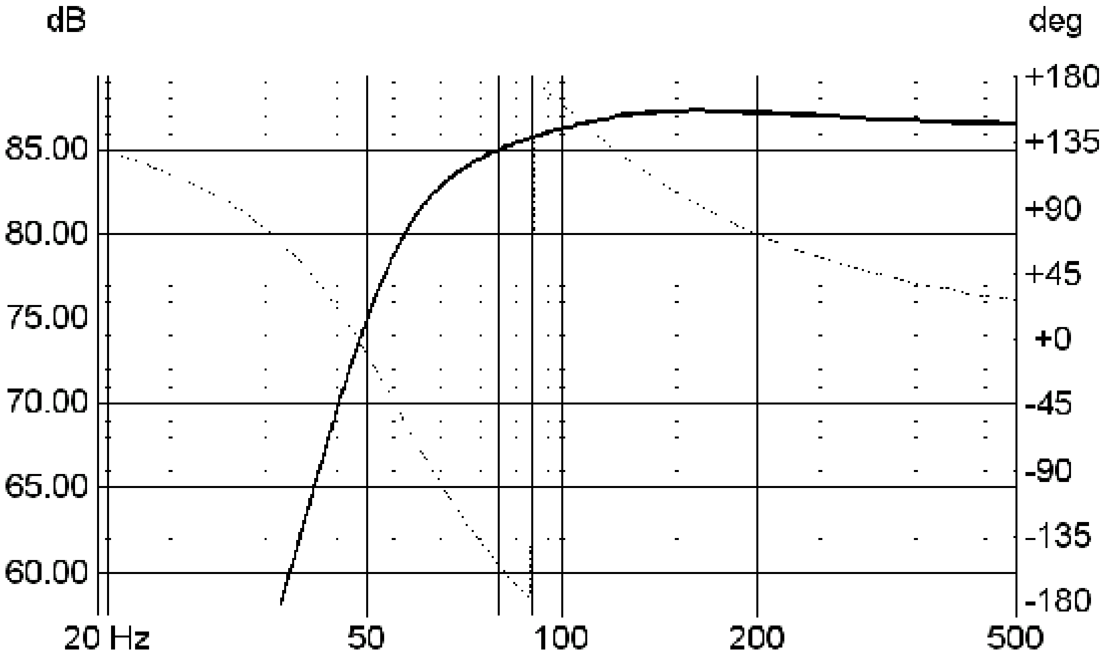

FIGURE 11. THE EQUALIZED FREQUENCY RESPONSE,
FIGURE 11 = FIGURE 4 + FIGURE 10.

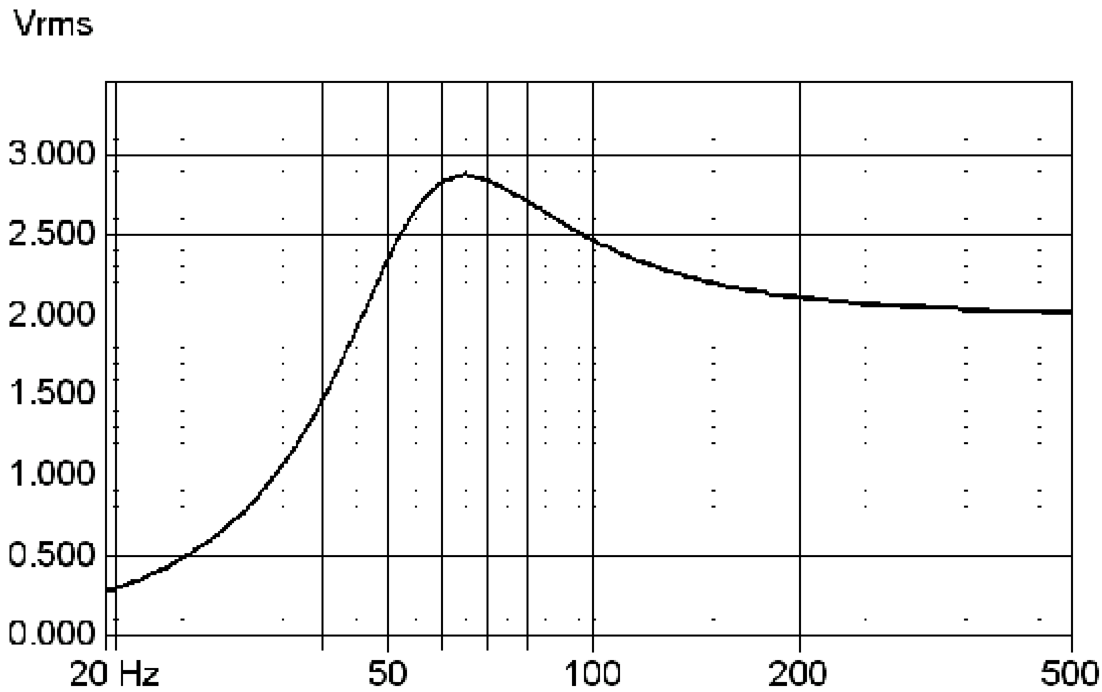

FIGURE 12. AMPLIFIER OUTPUT VOLTAGE VS. FREQUENCY FOR FIGURE 11, 2.83 V MAX

Well figure 11 looks pretty good for a woofer as defined in figure 1, yes a 5.25-inch in a 1.75 l box. However this loudspeaker needs some low-pass filtering to implement a crossover at approximately 170 Hz. Will this impact the woofer's response?

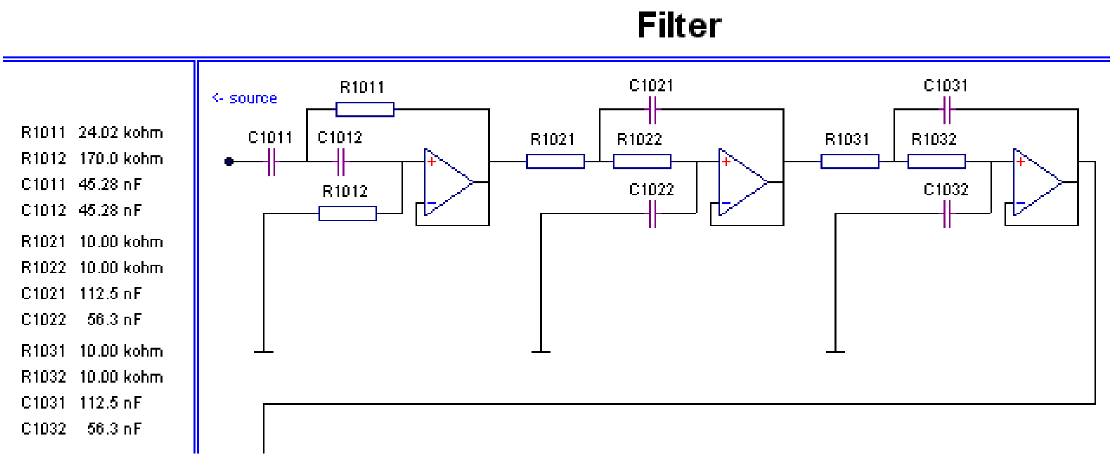

FIGURE 13. HP AND LP FILTERS' BLOCK DIAGRAM THAT WITH THE WOOFER IMPEMENT AN 8TH ORDER SYMMETRICAL BAND-PASS WOOFER.

Figure 13 illustrates the low level active filters used in this example, while figure 14 illustrates the filters' response verses frequency.

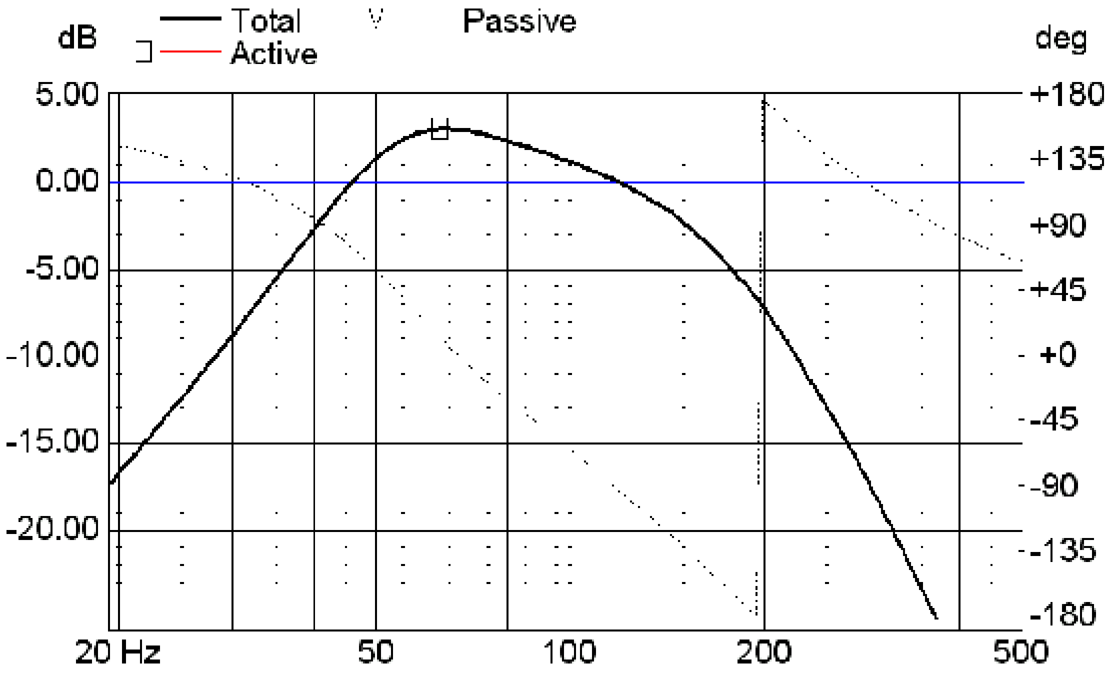

FIGURE 14. THE FILTERS' RESPONSE.

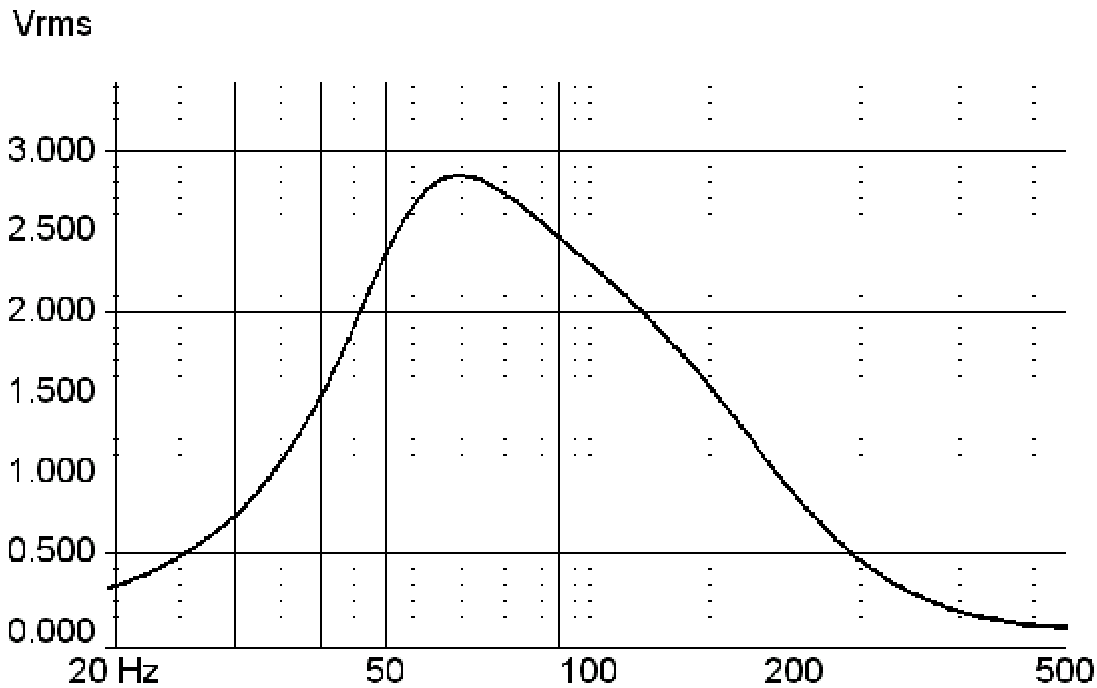

FIGURE 15. THE AMPLIFIER'S OUTPUT, 2.83 VOLTS MAX.

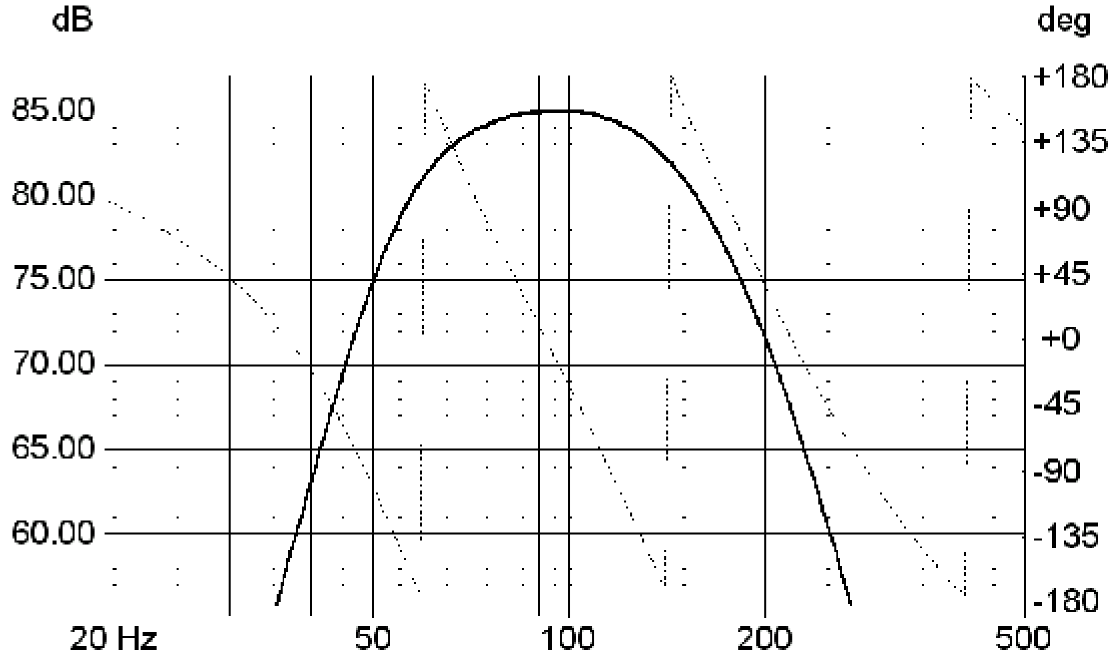

FIGURE 16. EQUALIZED 1.75 l SEALED BOX FREQUENCY RESPONSE SIMULATION WITH 85 dB SENSITIVITY AT 1.0 m WITH 2.83 VOLTS INPUT, AN 8TH ORDER SYMMETRICAL BAND-PASS WOOFER.

PLEASE NOTE THAT FIGURE 16 = FIGURE 4 + FIGURE 14

Well that looks much better and the - 6 dB cut-offs are at ~55 Hz and ~170 Hz with 85 dB at 1.0 m with 2.83 V_{rms} input, see figure 16. That seems reasonable for a small system and there will be zero port noise. However, it is very important to understand that there are no ports resonating and all the acoustic output of this woofer is the result of DISPLACEMENT. Typically, an equalized sealed box such as is described in this discussion will be mechanically limited. The large displacement helps to reduce the steady state operating temperature of the voice coil. Unlike the ported box, there is no blocked cone effect or displacement null at the tuning frequency.

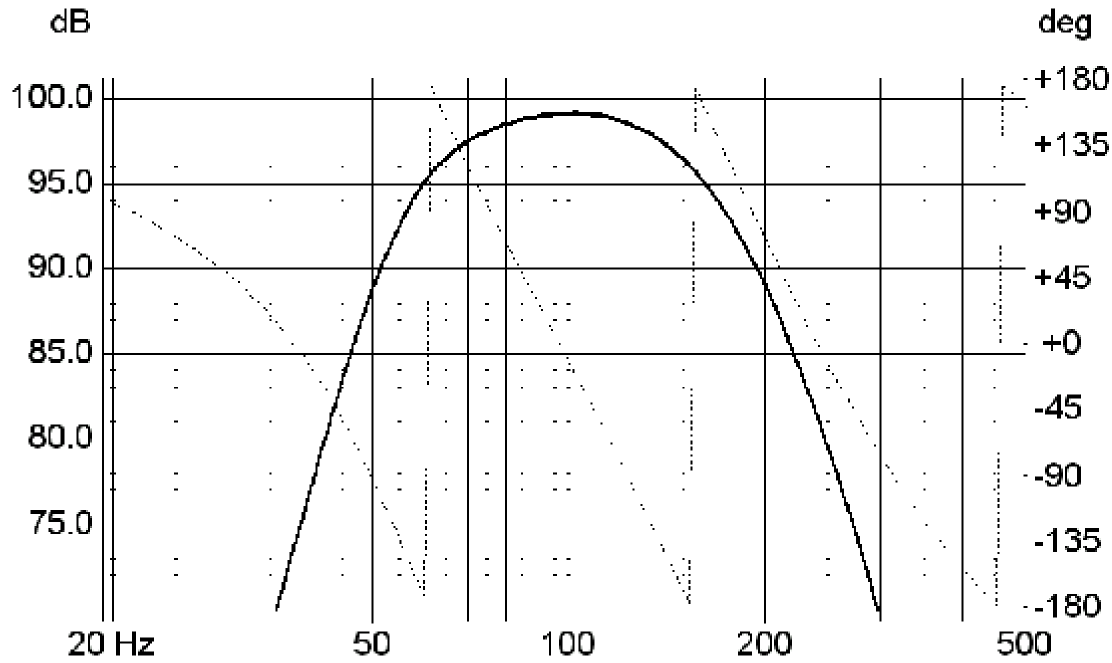

FIGURE 17. EQUALIZED 1.75 l SEALED BOX FREQUENCY RESPONSE SIMULATION WITH 85 dB SENSITIVITY AT 1.0 m WITH 14.1 VOLTS INPUT.

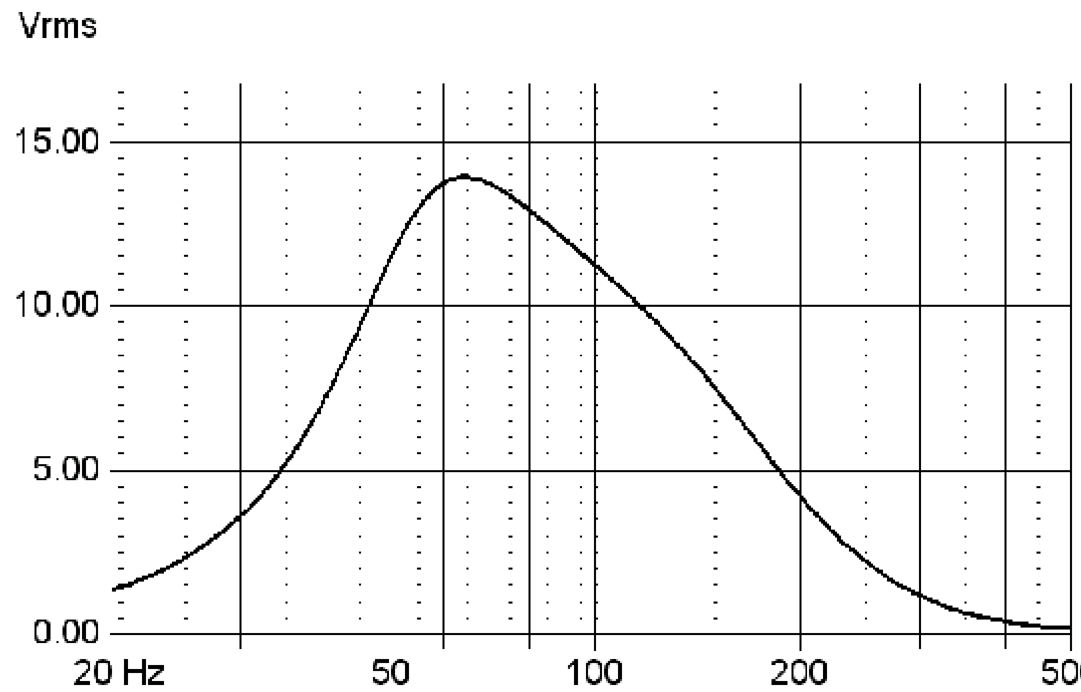

FIGURE 18. 50 W AMPLIFIER OUTPUT, 14.1 VOLTS INTO 4 Ω

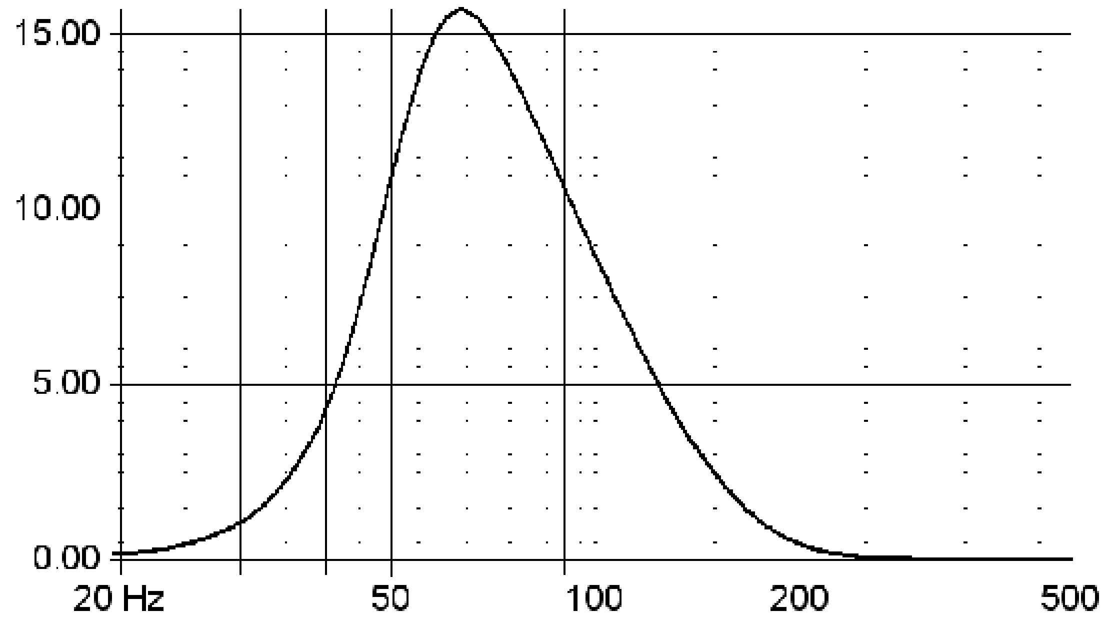

FIGURE 19. PEAK-PEAK DISPLACEMENT REQUIREMENT FOR NO MECHANICAL COMPRESSION (LINEAR MODEL) AT 50 W.

In the limit, the displacement illustrated in figure 18 depends on stiffness and drive force, where $\mathbf{x} = \frac{\mathbf{Bl}\mathbf{I}}{\mathbf{K}}$ (m). $\mathbf{I}$ is the input current (A). Figure 17 illustrates the amplifier's output voltage verses frequency. The magnitude of the input current is figure 17 ÷ figure 3.

Regardless of the enclosure design, a high-pass "garbage" filter as it is commonly referred to and with its cut-off below f_{box} is highly recommended. A properly designed high-pass filter will also help to control maximum displacement when the woofer is overdriven and will reduce harmonic distortion. An interesting characteristic of a woofer with a band-pass response like is illustrated in figure 16 is that most harmonics are out of band. The high cut-off is only 170 Hz in this example but this can be even lower. Now that the high-pass filter has also attenuated the very low frequency fundamentals, below 55 Hz, hopefully their harmonics are also effectively attenuated.

It may take a bit of thought to realize that the lesson here is for a sealed box woofer the output is essentially related to the area of the diaphragm, $\mathbf{S_d}$ (m^2) times the displacement, **x** (m). Having said that, a 5.25 inch woofer is small and a cut-off of 30 Hz is not realistic regardless of the transducer and/or the enclosure design but hopefully most designers know that. It's dangerous down there; displacements want to be very high for a $77cm^2$ diaphragm at 30 Hz.

One point that has not been discussed is that a reasonably high β is require for a suitable woofer for an equalized sealed box application and the pass band sensitivity goes as $10\log_{10}(\beta)$. All this means is that if β is doubled, sensitivity is increased by 3 dB, while if β is halved sensitivity is reduced by 3 dB. Without a high-pass filter, there are reliability concerns and questions regarding system warranty issues.

The downside is the time domain behavior. Any 8^{th} order system will exhibit energy storage and release. The sampling theory says that narrow band frequency response maps into ringing in the time domain. A perfect and wide bandwidth, DC to light in the frequency domain maps into the perfect impulse response in the time domain, the Dirac Delta or impulse function.

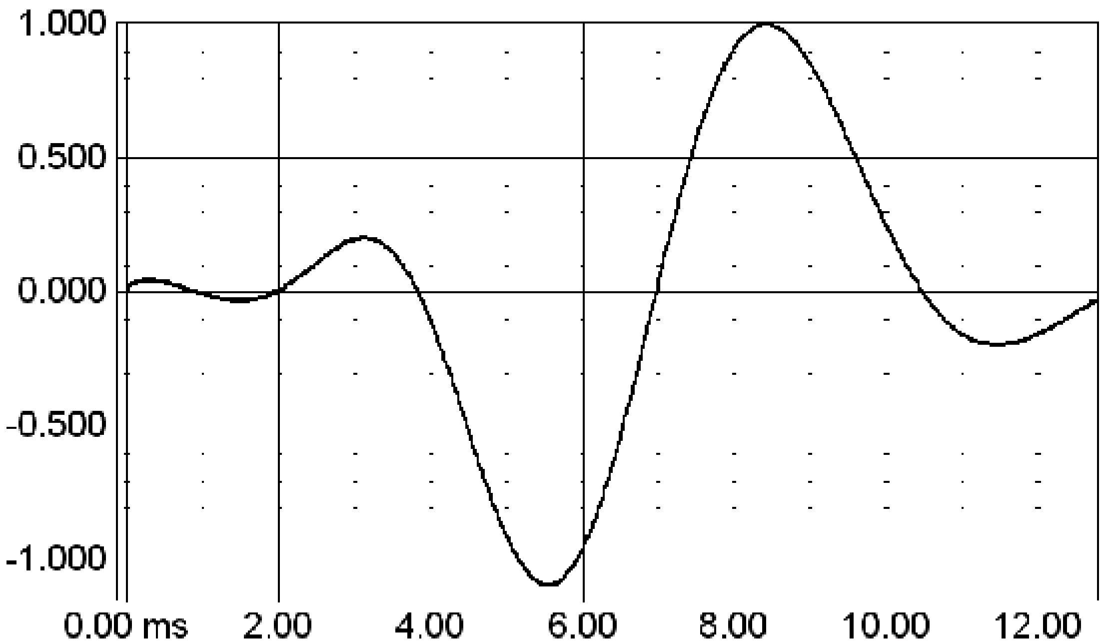

FIGURE 20. THE SYSTEM IMPULSE RESPONSE.

Please remember that the frequency response in figure 16 is exactly the Fourier transform of the impulse response illustrated in figure 20 above by definition! The transducer, the box and the filters implement a system or in this example model, a filter.

In closing, it should be mentioned that β, bandwidth, displacement and amplifier power must be compatible. There is not a unique solution to the small-equalized sealed box woofer loudspeaker but there is clearly a range of acceptable and/or desirable woofer characteristics. These are typically high β and large displacement capability. If stiffness is halved sensitivity is not effected; however, if mass is doubled, expect a reduction in sensitivity of 6 dB while the $\boldsymbol{f_o}$ will then only be reduced only by a factor of $\frac{1}{\sqrt{2}} \approx 0.7$.

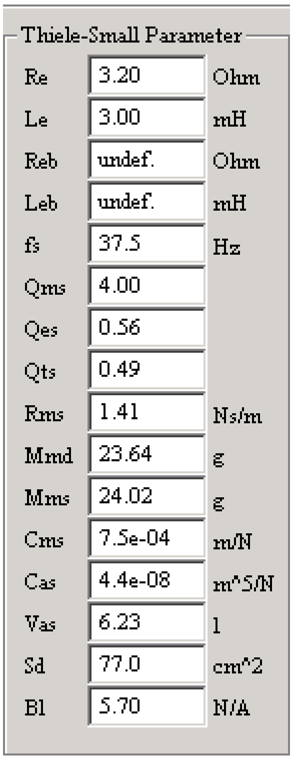

FIGURE 21. WOOFER MODEL, $\boldsymbol{f_o}$ = 37.5 Hz, mass = $\mathbf{2M_{md}}$.

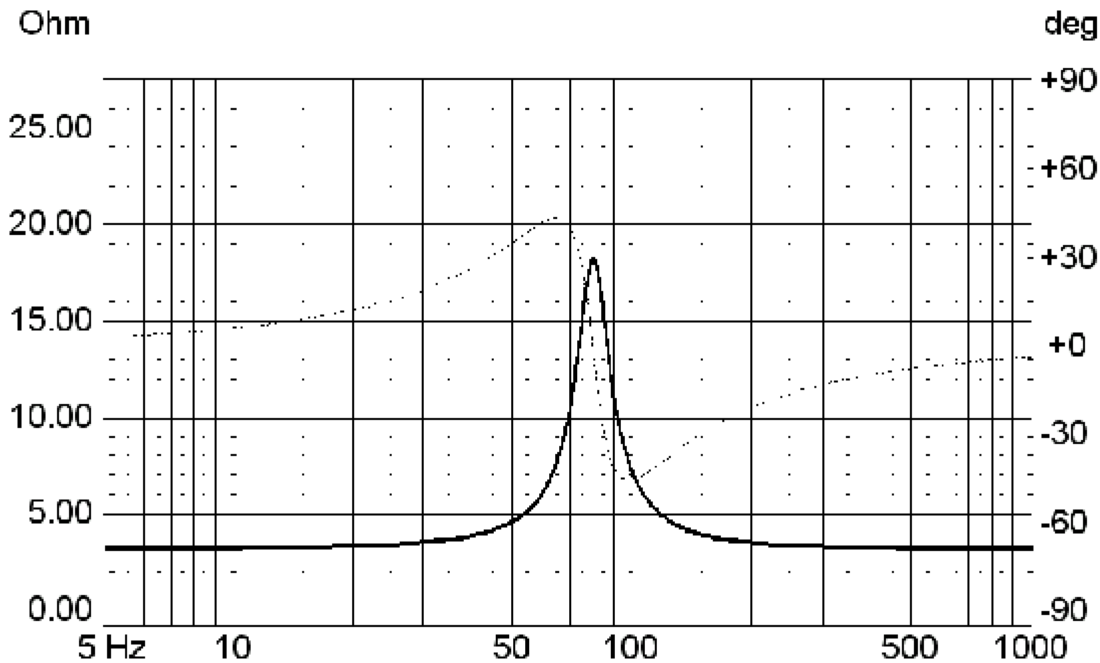

FIGURE 22. IMPEDANCE CURVE FOR THE HIGH MASS WOOFER MOUNTED IN A 1.8 l BOX, $\boldsymbol{f_{box} \approx 2f_o}$.

The box stiffness is still dominating but now the mass has been doubled.

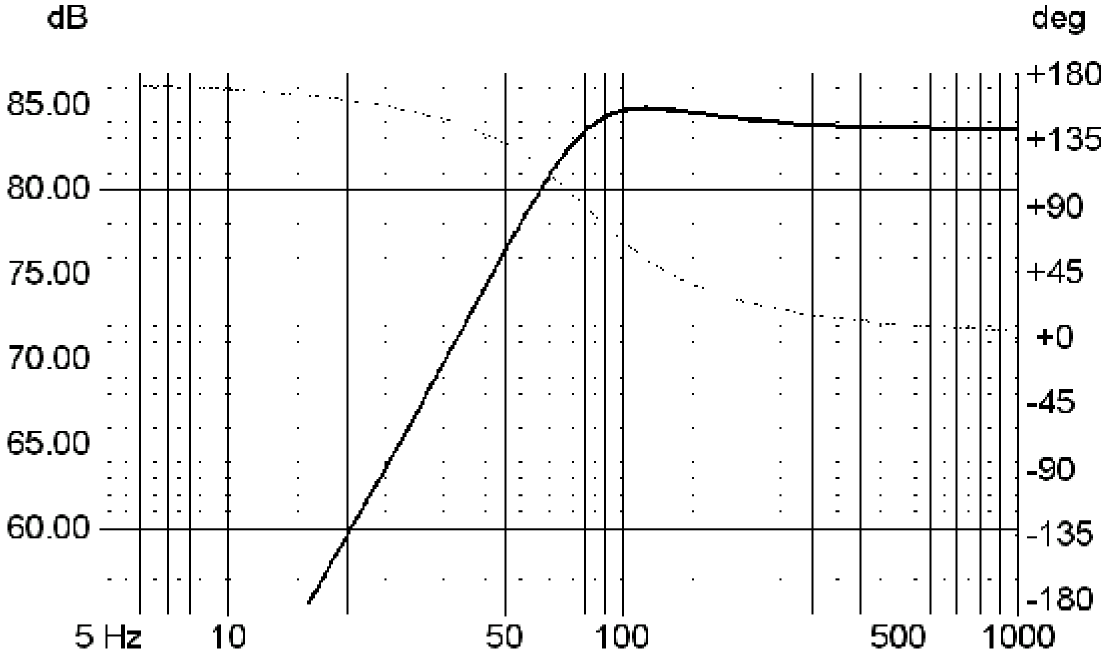

FIGURE 23. UNEQUALIZED FREQUQNCY RESPONSE FOR 37.5 HZ HIGH MASS WOOFER IN A 1.8 l SEALED BOX.

The response in figure 22 is less than 85 dB at 100 Hz and is a just noticeable difference relative to figures 4 and 9 but now the response is under-damped.

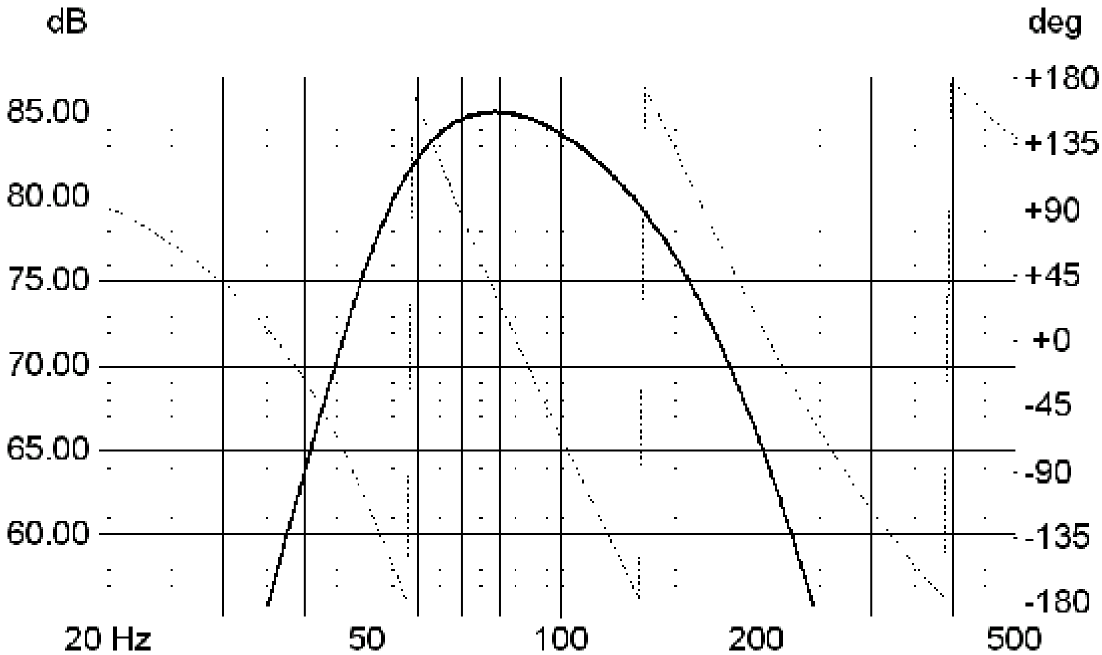

FIGURE 24. EQUALIZED 1.75 l SEALED BOX FREQUENCY RESPONSE SIMULATION AT 1 M WITH 2.83 V INPUT.

AGAIN NOTE THAT FIGURE 24 = FIGURE 23 + FIGURE 14

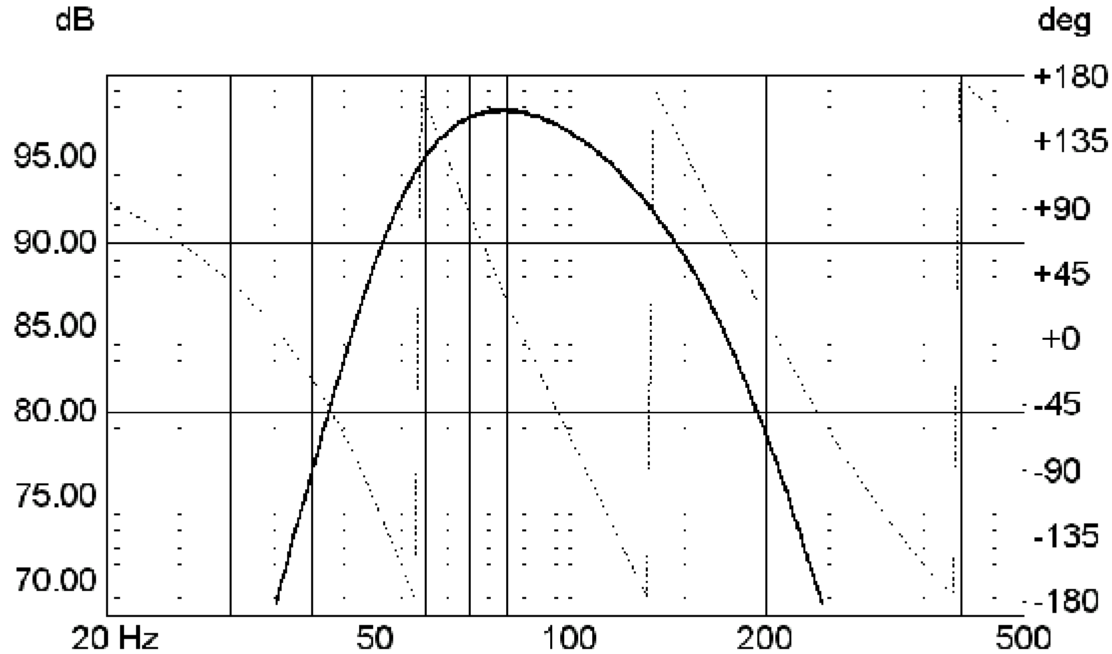

FIGURE 25. EQUALIZED 1.75 l SEALED BOX FREQUENCY RESPONSE SIMULATION AT 16 MM PEAK - PEAK DISPLACEMENT.

Doubling the mass has very little affect on the frequency response inclusive of bandwidth; see figures 16 and 24. Increasing mass with such a small box does not appear to be helpful and could induce mechanical stability issues at such high displacement. It seems that the transducer parameter targets should be to minimize mass and suspension stiffness balanced with a reasonable level of soft-part assembly robustness. Thus it was shown that f_o is not important in a small-equalized sealed box woofer.

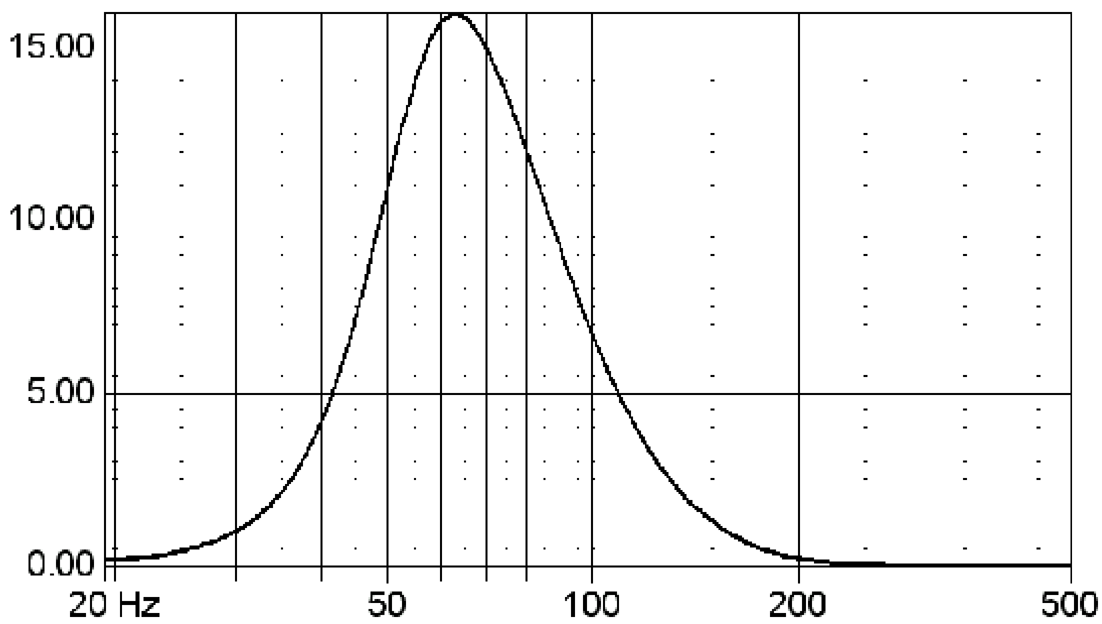

FIGURE 26. PEAK-PEAK DISPLACEMENT AT THE MECHANICAL LIMIT BUT AT ONLY 37 W INPUT.

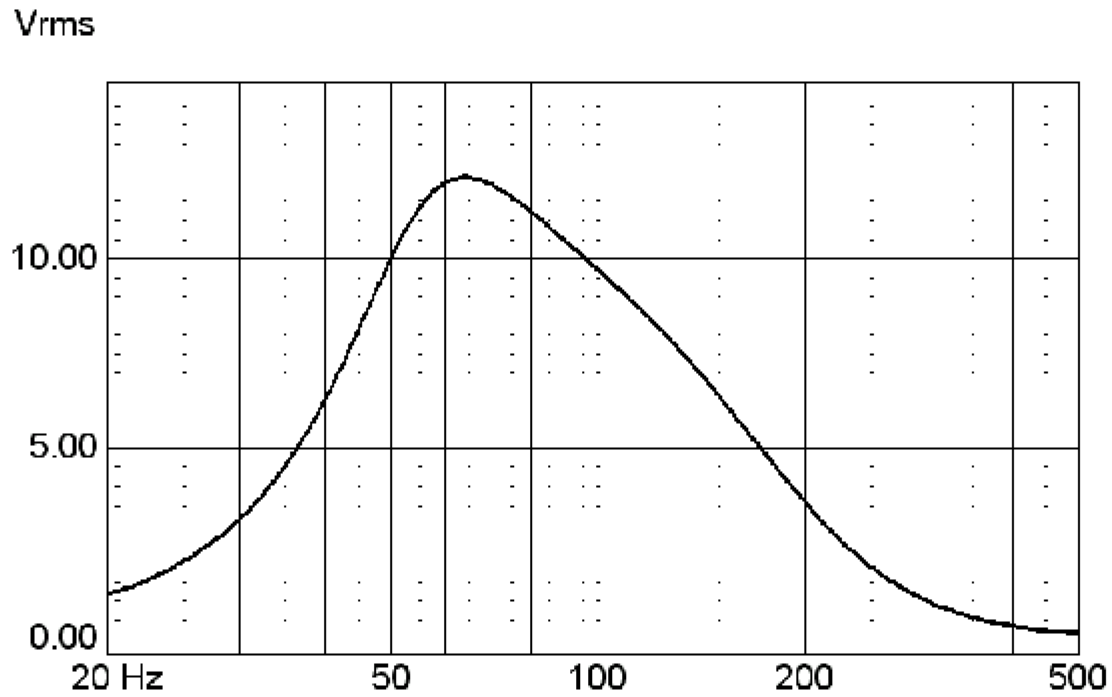

FIGURE 27. 37 W

AMPLIFIER OUTPUT, 12.1 VOLTS INTO 4 Ω

The system is displacement limited at 8 mm peak or 16 mm peak to peak. See figures 26 and 27. The dominant parameters are the box volume, β and the displacement, **x**.

Small loudspeakers with small boxes are beautiful but need filters and equalization if cut-offs of 60 Hz or below are the target.

Steve Mowry

3 June 2006

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